

数値解析 (塩田)

— ルジャンドル多項式 —

- $P_0(x) = 1$
- $P_1(x) = x$
実根 $\{t_k\}$: $\{0.\}$
重み $\{c_k\}$: $\{2.\}$
- $P_2(x) = -\frac{1}{2} + \frac{3x^2}{2}$
実根 $\{t_k\}$: $\{-0.577350269189626, 0.577350269189626\}$
重み $\{c_k\}$: $\{1., 1.\}$
- $P_3(x) = \frac{-3x}{2} + \frac{5x^3}{2}$
実根 $\{t_k\}$: $\{-0.774596669241483, 0., 0.774596669241483\}$
重み $\{c_k\}$: $\{0.555555555555556, 0.888888888888889, 0.555555555555556\}$
- $P_4(x) = \frac{3}{8} - \frac{15x^2}{4} + \frac{35x^4}{8}$
実根 $\{t_k\}$: $\{-0.861136311594053, -0.339981043584856, 0.339981043584856, 0.861136311594053\}$
重み $\{c_k\}$: $\{0.347854845137454, 0.652145154862546, 0.652145154862546, 0.347854845137454\}$
- $P_5(x) = \frac{15x}{8} - \frac{35x^3}{4} + \frac{63x^5}{8}$
実根 $\{t_k\}$: $\{-0.906179845938664, -0.538469310105683, 0., 0.538469310105683, 0.906179845938664\}$
重み $\{c_k\}$: $\{0.236926885056189, 0.478628670499366, 0.568888888888889, 0.478628670499366, 0.236926885056189\}$
- $P_6(x) = -\frac{5}{16} + \frac{105x^2}{16} - \frac{315x^4}{16} + \frac{231x^6}{16}$
実根 $\{t_k\}$: $\{-0.93247, -0.661209, -0.238619, 0.238619, 0.661209, 0.93247\}$
重み $\{c_k\}$: $\{0.171324, 0.360762, 0.467914, 0.467914, 0.360762, 0.171324\}$
- $P_7(x) = \frac{-35x}{16} + \frac{315x^3}{16} - \frac{693x^5}{16} + \frac{429x^7}{16}$
実根 $\{t_k\}$: $\{-0.949108, -0.741531, -0.405845, 0., 0.405845, 0.741531, 0.949108\}$
重み $\{c_k\}$: $\{0.129485, 0.279705, 0.38183, 0.417959, 0.38183, 0.279705, 0.129485\}$
- $P_8(x) = \frac{35}{128} - \frac{315x^2}{32} + \frac{3465x^4}{64} - \frac{3003x^6}{32} + \frac{6435x^8}{128}$
実根 $\{t_k\}$: $\{-0.96029, -0.796666, -0.525532, -0.183435, 0.183435, 0.525532, 0.796666, 0.96029\}$
重み $\{c_k\}$: $\{0.101229, 0.222381, 0.313707, 0.362684, 0.362684, 0.313707, 0.222381, 0.101229\}$
- $P_9(x) = \frac{315x}{128} - \frac{1155x^3}{32} + \frac{9009x^5}{64} - \frac{6435x^7}{32} + \frac{12155x^9}{128}$
実根 $\{t_k\}$: $\{-0.96816, -0.836031, -0.613371, -0.324253, 0., 0.324253, 0.613371, 0.836031, 0.96816\}$
重み $\{c_k\}$: $\{0.0812744, 0.180648, 0.260611, 0.312347, 0.330239, 0.312347, 0.260611, 0.180648, 0.0812744\}$
- $P_{10}(x) = -\frac{63}{256} + \frac{3465x^2}{256} - \frac{15015x^4}{128} + \frac{45045x^6}{128} - \frac{109395x^8}{256} + \frac{46189x^{10}}{256}$
実根 $\{t_k\}$: $\{-0.973907, -0.865063, -0.67941, -0.433395, -0.148874, 0.148874, 0.433395, 0.67941, 0.865063, 0.973907\}$
重み $\{c_k\}$: $\{0.0666713, 0.149451, 0.219086, 0.269267, 0.295524, 0.295524, 0.269267, 0.219086, 0.149451, 0.0666713\}$

x	$P_0(x)$	$P_1(x)$	$P_2(x)$	$P_3(x)$	$P_4(x)$	$P_5(x)$
-1	1	-1	1	-1	1	-1
-0.95	1	-0.95	0.85375	-0.7184	0.55409	-0.3727
-0.9	1	-0.9	0.715	-0.4725	0.20794	0.04114
-0.85	1	-0.85	0.58375	-0.2603	-0.0506	0.28566
-0.8	1	-0.8	0.46	-0.08	-0.233	0.39952
-0.75	1	-0.75	0.34375	0.07031	-0.3501	0.41638
-0.7	1	-0.7	0.235	0.1925	-0.4121	0.3652
-0.65	1	-0.65	0.13375	0.28844	-0.4284	0.27049
-0.6	1	-0.6	0.04	0.36	-0.408	0.15264
-0.55	1	-0.55	-0.0462	0.40906	-0.359	0.02819
-0.5	1	-0.5	-0.125	0.4375	-0.2891	-0.0898
-0.45	1	-0.45	-0.1963	0.44719	-0.205	-0.1917
-0.4	1	-0.4	-0.26	0.44	-0.113	-0.2706
-0.35	1	-0.35	-0.3163	0.41781	-0.0187	-0.3225
-0.3	1	-0.3	-0.365	0.3825	0.07294	-0.3454
-0.25	1	-0.25	-0.4063	0.33594	0.15771	-0.3397
-0.2	1	-0.2	-0.44	0.28	0.232	-0.3075
-0.15	1	-0.15	-0.4663	0.21656	0.29284	-0.2523
-0.1	1	-0.1	-0.485	0.1475	0.33794	-0.1788
-0.05	1	-0.05	-0.4963	0.07469	0.36565	-0.0927
0	1	0	-0.5	0	0.375	0
0.05	1	0.05	-0.4963	-0.0747	0.36565	0.09266
0.1	1	0.1	-0.485	-0.1475	0.33794	0.17883
0.15	1	0.15	-0.4663	-0.2166	0.29284	0.25232
0.2	1	0.2	-0.44	-0.28	0.232	0.30752
0.25	1	0.25	-0.4063	-0.3359	0.15771	0.33972
0.3	1	0.3	-0.365	-0.3825	0.07294	0.34539
0.35	1	0.35	-0.3163	-0.4178	-0.0187	0.32245
0.4	1	0.4	-0.26	-0.44	-0.113	0.27064
0.45	1	0.45	-0.1963	-0.4472	-0.205	0.19172
0.5	1	0.5	-0.125	-0.4375	-0.2891	0.08984
0.55	1	0.55	-0.0462	-0.4091	-0.359	-0.0282
0.6	1	0.6	0.04	-0.36	-0.408	-0.1526
0.65	1	0.65	0.13375	-0.2884	-0.4284	-0.2705
0.7	1	0.7	0.235	-0.1925	-0.4121	-0.3652
0.75	1	0.75	0.34375	-0.0703	-0.3501	-0.4164
0.8	1	0.8	0.46	0.08	-0.233	-0.3995
0.85	1	0.85	0.58375	0.26031	-0.0506	-0.2857
0.9	1	0.9	0.715	0.4725	0.20794	-0.0411
0.95	1	0.95	0.85375	0.71844	0.55409	0.37274
1	1	1	1	1	1	1

