

数值解析 (塩田)

— 数值微分 —

中心差分

$$\frac{\delta y}{\delta x} = \frac{1}{2h} \{y(x+h) - y(x-h)\}$$

$$\frac{\delta^2 y}{\delta x^2} = \frac{1}{h^2} \{y(x+h) - 2y(x) + y(x-h)\}$$

$$\frac{\delta^3 y}{\delta x^3} = \frac{1}{2h^3} \{y(x+2h) - 2y(x+h) + 2y(x-h) - y(x-2h)\}$$

$$\frac{\delta^4 y}{\delta x^4} = \frac{1}{h^4} \{y(x+2h) - 4y(x+h) + 6y(x) - 4y(x-h) + y(x-2h)\}$$

$$\frac{\delta^5 y}{\delta x^5} = \frac{1}{2h^5} \{y(x+3h) - 4y(x+2h) + 5y(x+h) - 5y(x-h) + 4y(x-2h) - y(x-3h)\}$$

$$\frac{\delta^6 y}{\delta x^6} = \frac{1}{h^6} \{y(x+3h) - 6y(x+2h) + 15y(x+h) - 20y(x) + 15y(x-h) - 6y(x-2h) + y(x-3h)\}$$

$$\frac{\delta^7 y}{\delta x^7} = \frac{1}{2h^7} \{y(x+4h) - 6y(x+3h) + 14y(x+2h) - 14y(x+h) + 14y(x-h) - 6y(x-2h) + y(x-4h)\}$$

$$\frac{\delta^8 y}{\delta x^8} = \frac{1}{h^8} \{y(x+4h) - 8y(x+3h) + 28y(x+2h) - 56y(x+h) + 70y(x) - 56y(x-h) + 28y(x-2h) - 8y(x-3h) + y(x-4h)\}$$

中心差分を用いた補正公式

$$y'(x) = \frac{\delta y}{\delta x} - \frac{1}{6} \frac{\delta^3 y}{\delta x^3} h^2 + \frac{1}{30} \frac{\delta^5 y}{\delta x^5} h^4 - \frac{1}{140} \frac{\delta^7 y}{\delta x^7} h^6 + \dots$$

$$y''(x) = \frac{\delta^2 y}{\delta x^2} - \frac{1}{12} \frac{\delta^4 y}{\delta x^4} h^2 + \frac{1}{90} \frac{\delta^6 y}{\delta x^6} h^4 - \frac{1}{560} \frac{\delta^8 y}{\delta x^8} h^6 + \dots$$

$$y^{(3)}(x) = \frac{\delta^3 y}{\delta x^3} - \frac{1}{4} \frac{\delta^5 y}{\delta x^5} h^2 + \frac{7}{120} \frac{\delta^7 y}{\delta x^7} h^4 - \dots$$

$$y^{(4)}(x) = \frac{\delta^4 y}{\delta x^4} - \frac{1}{6} \frac{\delta^6 y}{\delta x^6} h^2 + \frac{7}{240} \frac{\delta^8 y}{\delta x^8} h^4 - \dots$$

前進差分

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{1}{h} \{y(x+h) - y(x)\} \\ \frac{\Delta^2 y}{\Delta x^2} &= \frac{1}{h^2} \{y(x+2h) - 2y(x+h) + y(x)\} \\ \frac{\Delta^3 y}{\Delta x^3} &= \frac{1}{h^3} \{y(x+3h) - 3y(x+2h) + 3y(x+h) - y(x)\} \\ \frac{\Delta^4 y}{\Delta x^4} &= \frac{1}{h^4} \{y(x+4h) - 4y(x+3h) + 6y(x+2h) - 4y(x+h) + y(x)\} \\ \frac{\Delta^5 y}{\Delta x^5} &= \frac{1}{h^5} \{y(x+5h) - 5y(x+4h) + 10y(x+3h) - 10y(x+2h) \\ &\quad + 5y(x+h) - y(x)\} \\ \frac{\Delta^6 y}{\Delta x^6} &= \frac{1}{h^6} \{y(x+6h) - 6y(x+5h) + 15y(x+4h) - 20y(x+3h) \\ &\quad + 15y(x+2h) - 6y(x+h) + y(x)\} \\ \frac{\Delta^7 y}{\Delta x^7} &= \frac{1}{h^7} \{y(x+7h) - 7y(x+6h) + 21y(x+5h) - 35y(x+4h) \\ &\quad + 35y(x+3h) - 21y(x+2h) + 7y(x+h) - y(x)\} \\ \frac{\Delta^8 y}{\Delta x^8} &= \frac{1}{h^8} \{y(x+8h) - 8y(x+7h) + 28y(x+6h) - 56y(x+5h) \\ &\quad + 70y(x+4h) - 56y(x+3h) + 28y(x+2h) - 8y(x+h) + y(x)\}\end{aligned}$$

前進差分を用いた補正公式

$$\begin{aligned}y'(x) &= \frac{\Delta y}{\Delta x} - \frac{1}{2} \frac{\Delta^2 y}{\Delta x^2} h + \frac{1}{3} \frac{\Delta^3 y}{\Delta x^3} h^2 - \frac{1}{4} \frac{\Delta^4 y}{\Delta x^4} h^3 + \frac{1}{5} \frac{\Delta^5 y}{\Delta x^5} h^4 - \frac{1}{6} \frac{\Delta^6 y}{\Delta x^6} h^5 \\ &\quad + \frac{1}{7} \frac{\Delta^7 y}{\Delta x^7} h^6 - \frac{1}{8} \frac{\Delta^8 y}{\Delta x^8} h^7 + \dots \\ y''(x) &= \frac{\Delta^2 y}{\Delta x^2} - \frac{\Delta^3 y}{\Delta x^3} h + \frac{11}{12} \frac{\Delta^4 y}{\Delta x^4} h^2 - \frac{5}{6} \frac{\Delta^5 y}{\Delta x^5} h^3 + \frac{137}{180} \frac{\Delta^6 y}{\Delta x^6} h^4 - \frac{7}{10} \frac{\Delta^7 y}{\Delta x^7} h^5 \\ &\quad + \frac{363}{560} \frac{\Delta^8 y}{\Delta x^8} h^6 - \dots \\ y^{(3)}(x) &= \frac{\Delta^3 y}{\Delta x^3} - \frac{3}{2} \frac{\Delta^4 y}{\Delta x^4} h + \frac{7}{4} \frac{\Delta^5 y}{\Delta x^5} h^2 - \frac{15}{8} \frac{\Delta^6 y}{\Delta x^6} h^3 + \frac{29}{15} \frac{\Delta^7 y}{\Delta x^7} h^4 \\ &\quad - \frac{469}{240} \frac{\Delta^8 y}{\Delta x^8} h^5 + \dots \\ y^{(4)}(x) &= \frac{\Delta^4 y}{\Delta x^4} - 2 \frac{\Delta^5 y}{\Delta x^5} h + \frac{17}{6} \frac{\Delta^6 y}{\Delta x^6} h^2 - \frac{7}{2} \frac{\Delta^7 y}{\Delta x^7} h^3 + \frac{967}{240} \frac{\Delta^8 y}{\Delta x^8} h^4 - \dots\end{aligned}$$